Coriolis Impact on Wind Farm Wakes

Under what conditions will the Coriolis Force contribute to wake recovery?

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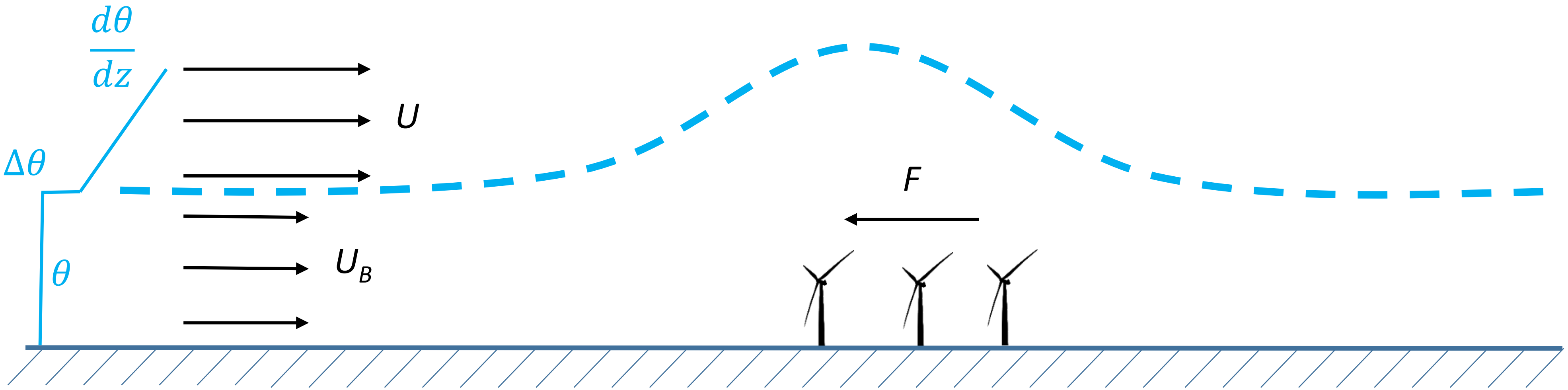
Interacting Hydrodynamic Processes

- Turbine drag
- Fluid inertia
- Lateral mixing of momentum
- Vertical mixing of momentum (represented as Rayleigh friction)
- Buoyancy waves on the inversion (creates a pressure field)
- Vertically propagating gravity waves (creates a pressure field)
- Lateral flow deflection and acceleration by Coriolis Force
- Inertial waves and recovery overshoot
- Geostrophic adjustment in the wake

Two-Layer Model (Side View)

- Two-layer model of Atmospheric Boundary Layer
  - Depth-averaged quantities



- Analytical solution for horizontal pressure field set up by ABL displacement
- Coupled to conservation of mass and momentum within ABL
- Closed equation set
- Linearisation around undisturbed condition permits very fast FFT solution

Governing Equation

$$U\frac{\partial u}{\partial x}+V\frac{\partial u}{\partial y}=F_x-\left(\frac{1}{\rho}\right)\frac{\partial p}{\partial x}-Cu+K\nabla^2 u+fv$$

$$U\frac{\partial v}{\partial x}+V\frac{\partial v}{\partial y}=F_y-\left(\frac{1}{\rho}\right)\frac{\partial p}{\partial y}-Cv+K\nabla^2 v-fu$$

Rayleigh Friction

Coriolis Force

Solution in Fourier Space

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$$\hat{u}(k,l)=\frac{D\widehat{F_x}+f\widehat{F_y}-i\Phi(Dk+fl)\hat{\eta}}{(D^2+f^2)}$$

$$\hat{v}(k,l)=\frac{D\widehat{F_y}-f\hat{F}_x-i\Phi(Dl-fk)\hat{\eta}}{(D^2+f^2)}$$

$$\hat{\eta}(k,l)=\frac{-H[k(D\widehat{F_x}+f\widehat{F_y})+l(D\widehat{F_y}-f\widehat{F_x})]}{\sigma(D^2+f^2)-iDH(k^2+l^2)\Phi}$$

$$D(k,l)=ikU+ilV+C+K(k^2+l^2)$$

Typical FFT Solution – Plan View

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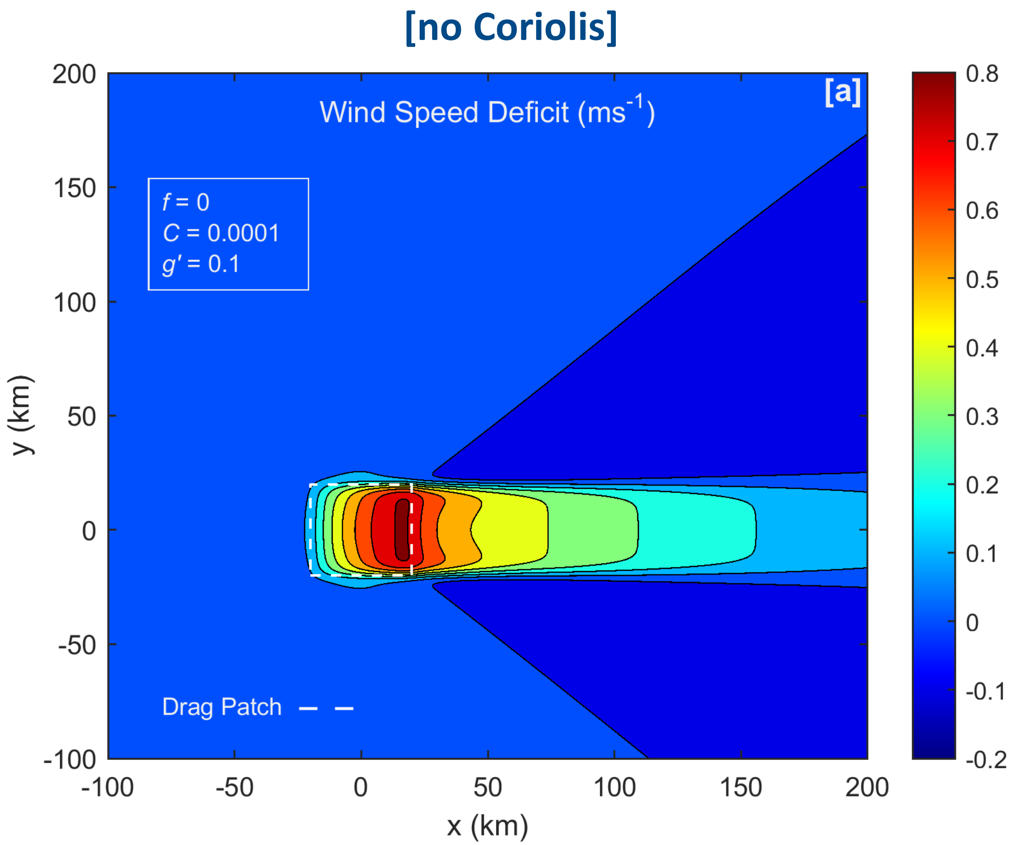
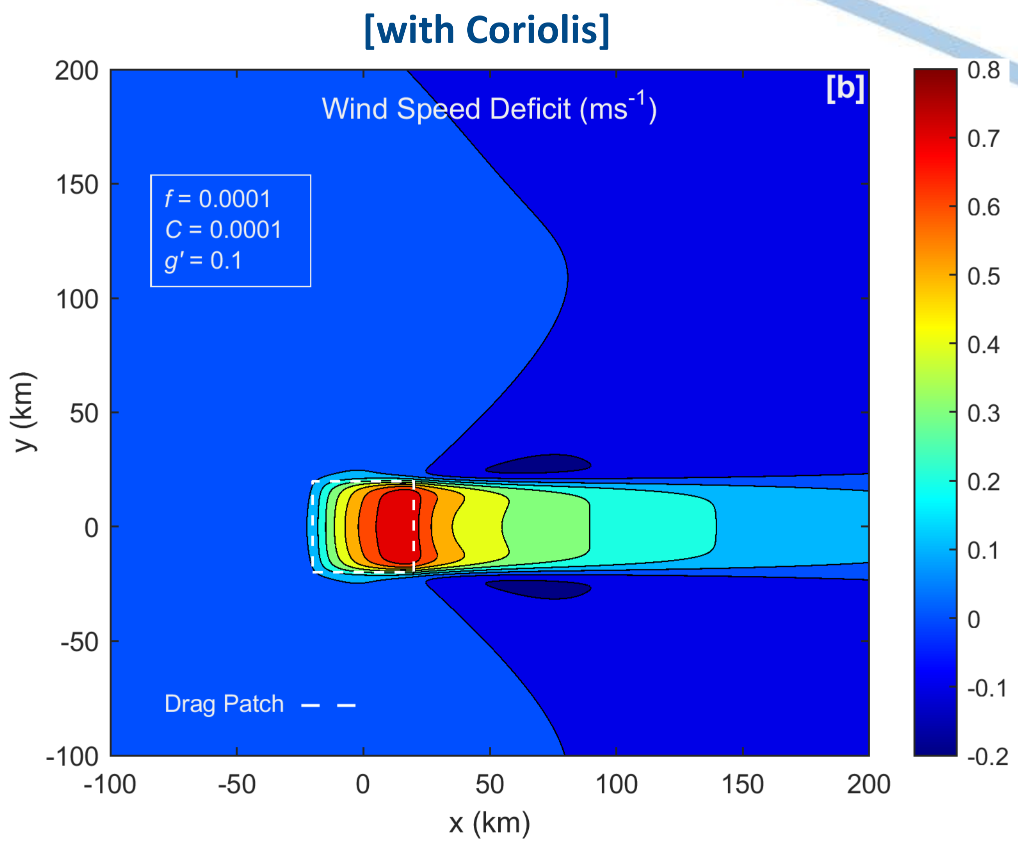
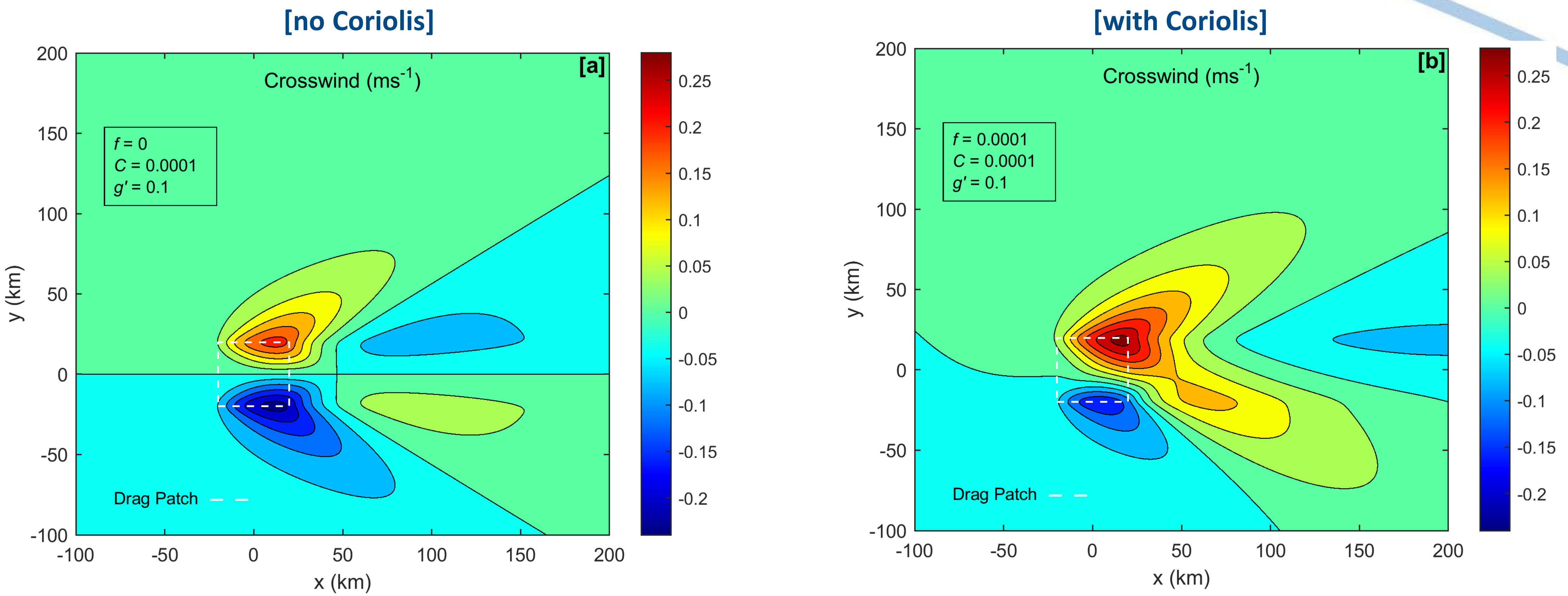




Speed Deficit (Stream-wise)

Typical FFT Solution – Plan View



Cross-Wind Speed

Typical FFT Solution – Plan View

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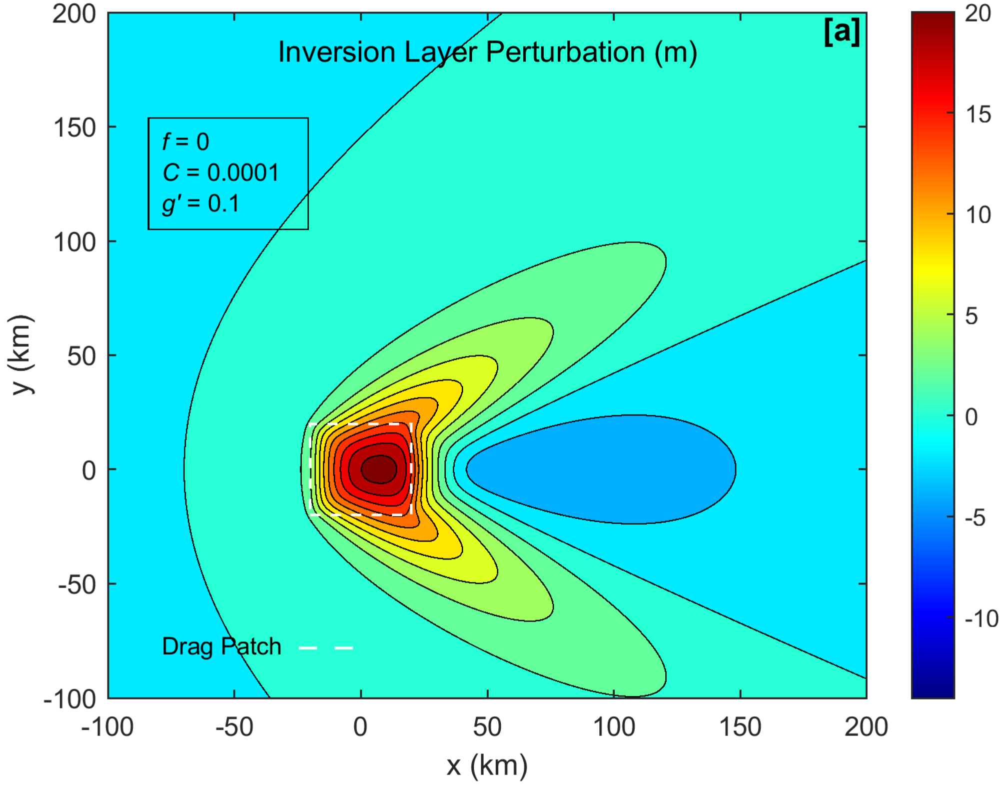
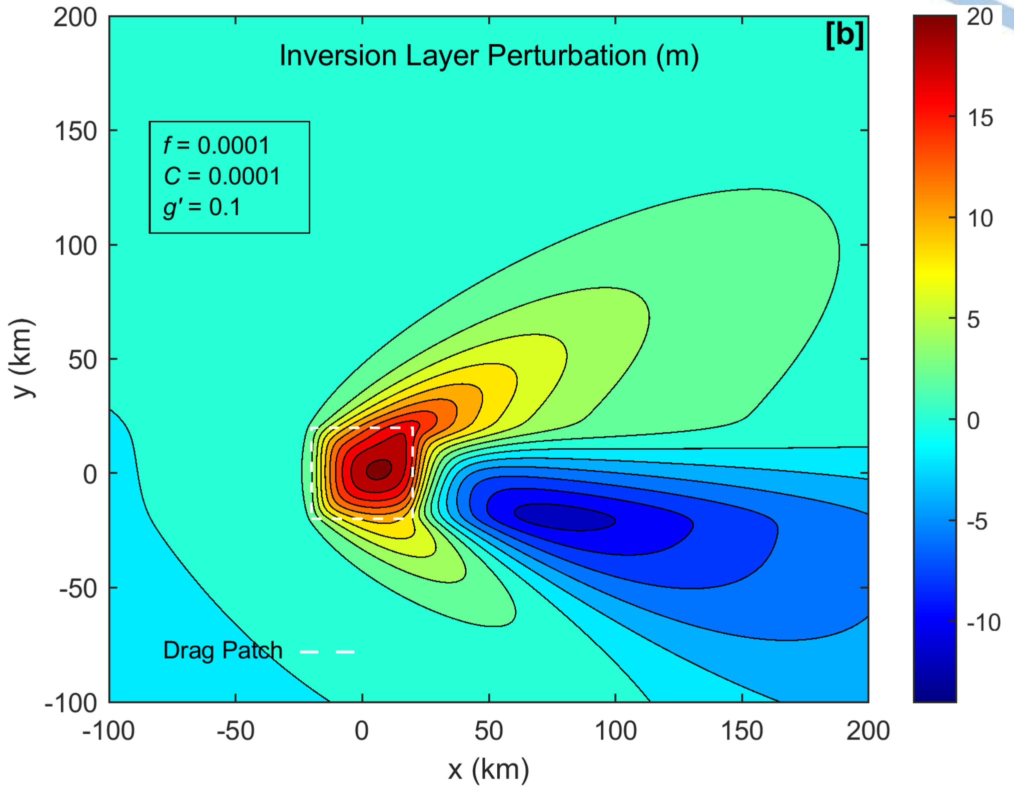
[no Coriolis]



[with Coriolis]



Vertical Displacement

Axial Forces along the Centreline

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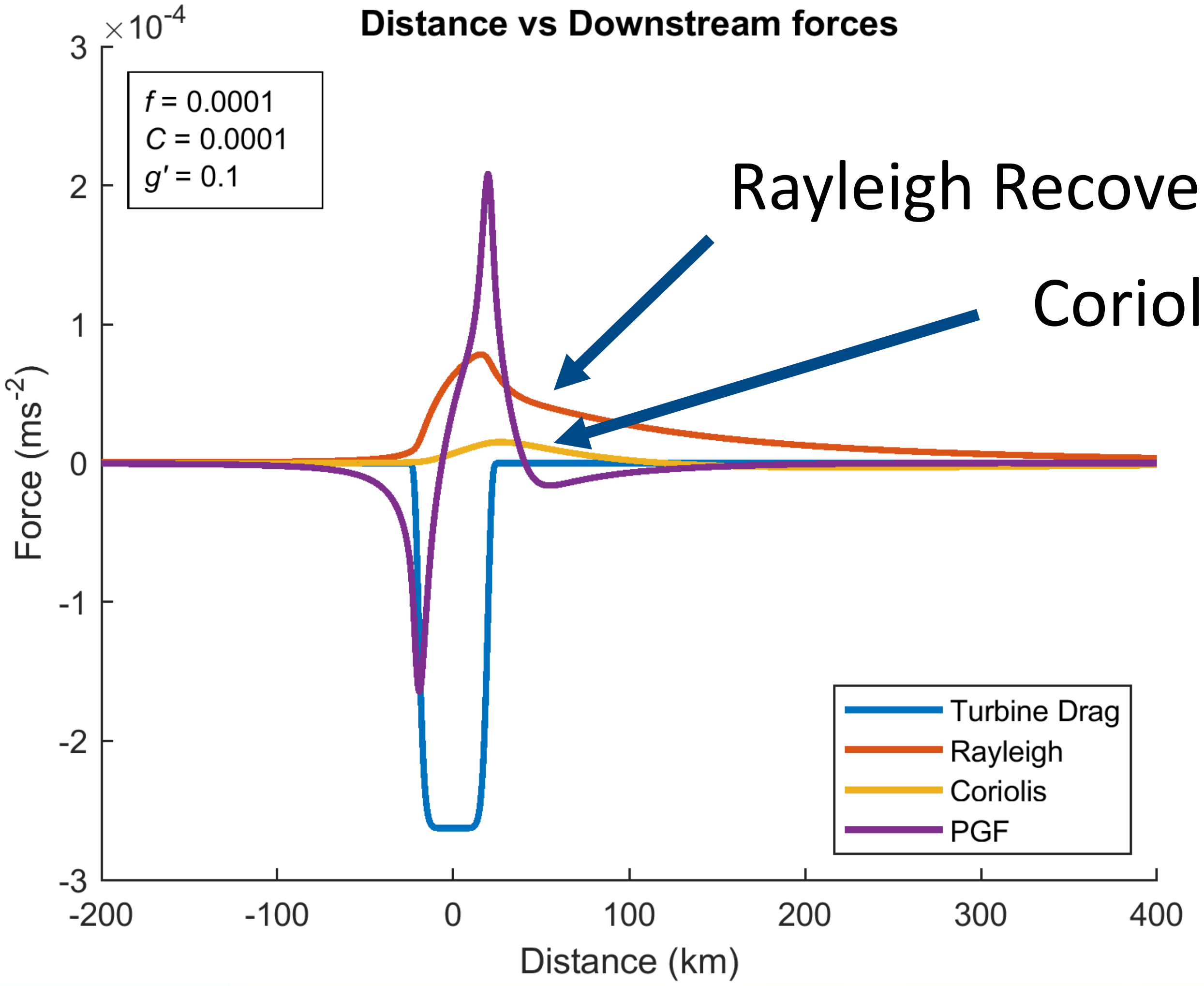
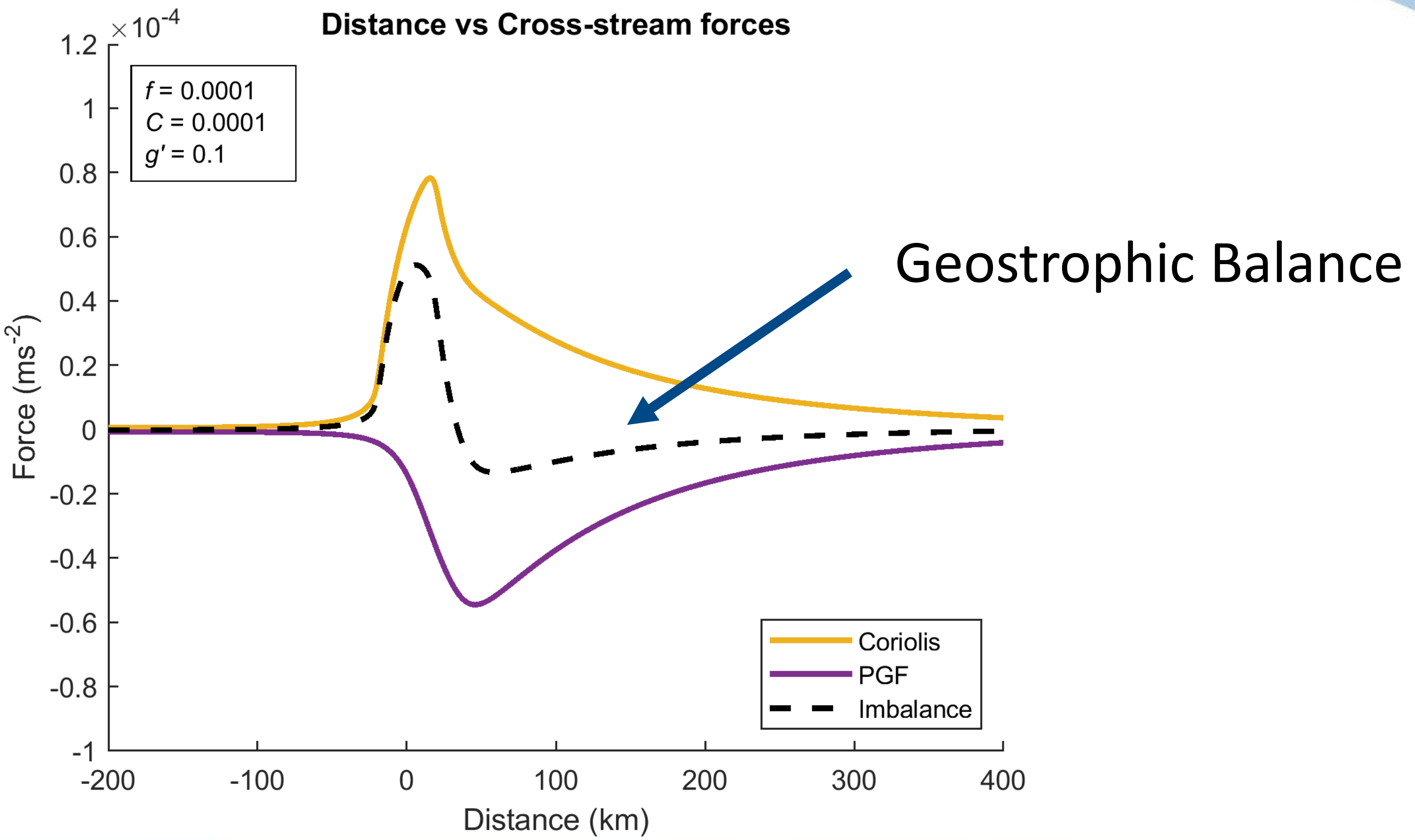


Rayleigh Recovery

Coriolis Recovery

Cross Forces along the Centreline



Global Fractional Coriolis Recovery (FCR) and Fractional Rayleigh Recovery (FRR)

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$$FCR = \frac{1}{1+(\frac{C}{f})^2} \qquad FRR = \frac{1}{1+(\frac{f}{C})^2}$$

Small C/f gives Coriolis more time to act

$$FCR \rightarrow 1$$

Centreline Fractional Coriolis Recovery (FCR)

Special Case
C=0

$$FCR = [1 - \exp(-\overline{FS})]$$

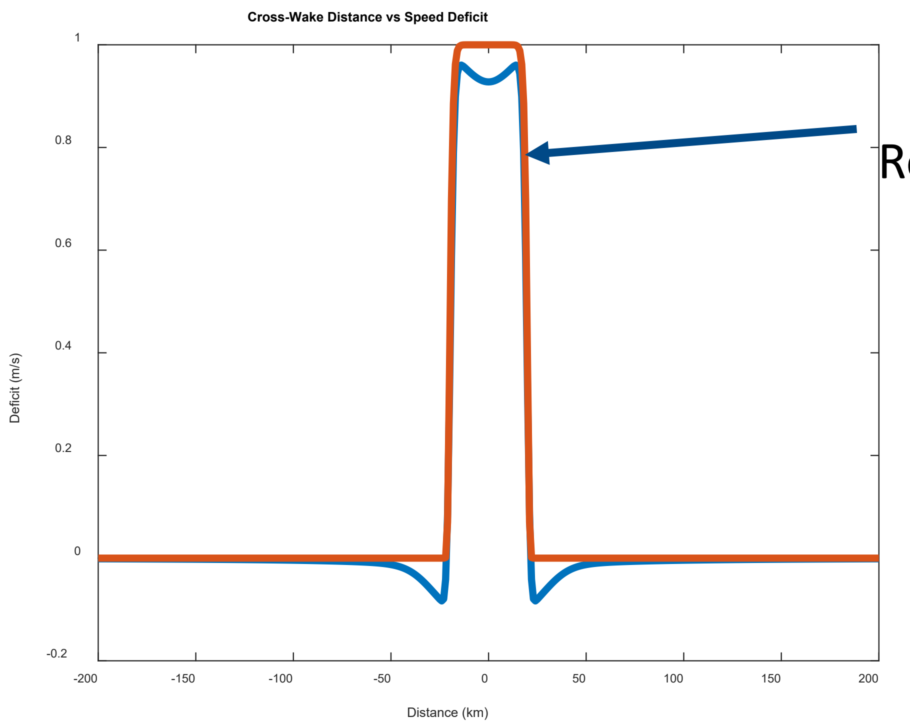
Non-dimension
Farm Width

$$\overline{FS} = af/\sqrt{g'H}$$

$$If \quad \overline{FS} \ll 1 \quad little\ FCR$$
$$If \quad \overline{FS} \gg 1 \quad strong\ FCR$$

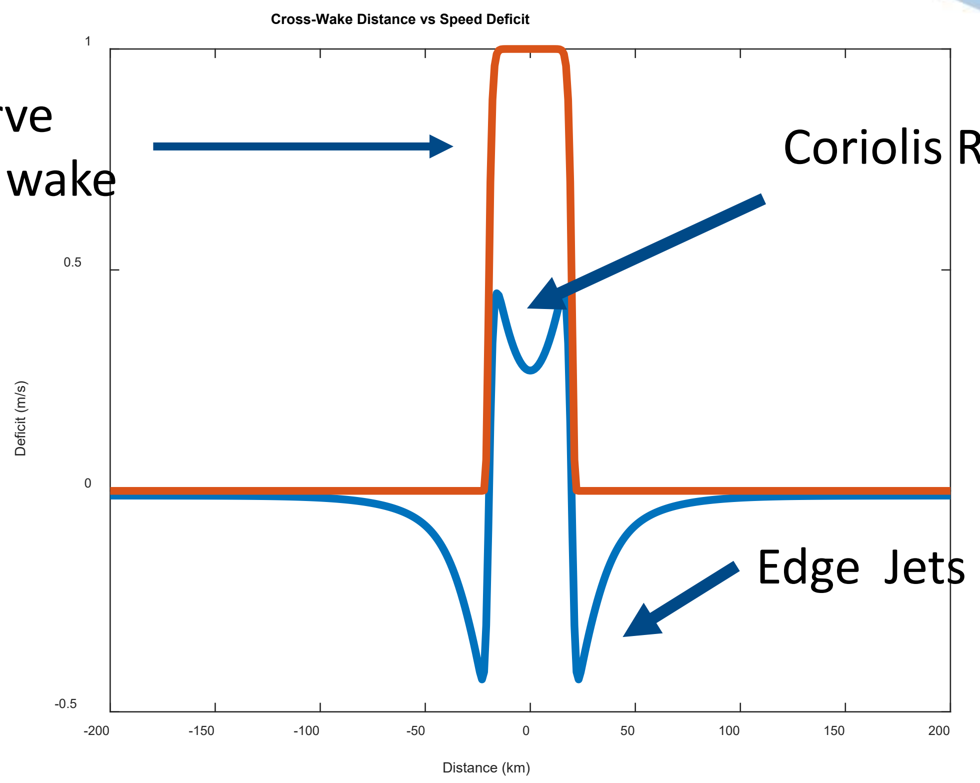
Wake Deficit Profile 60 km downwind

[no Coriolis, f=0]



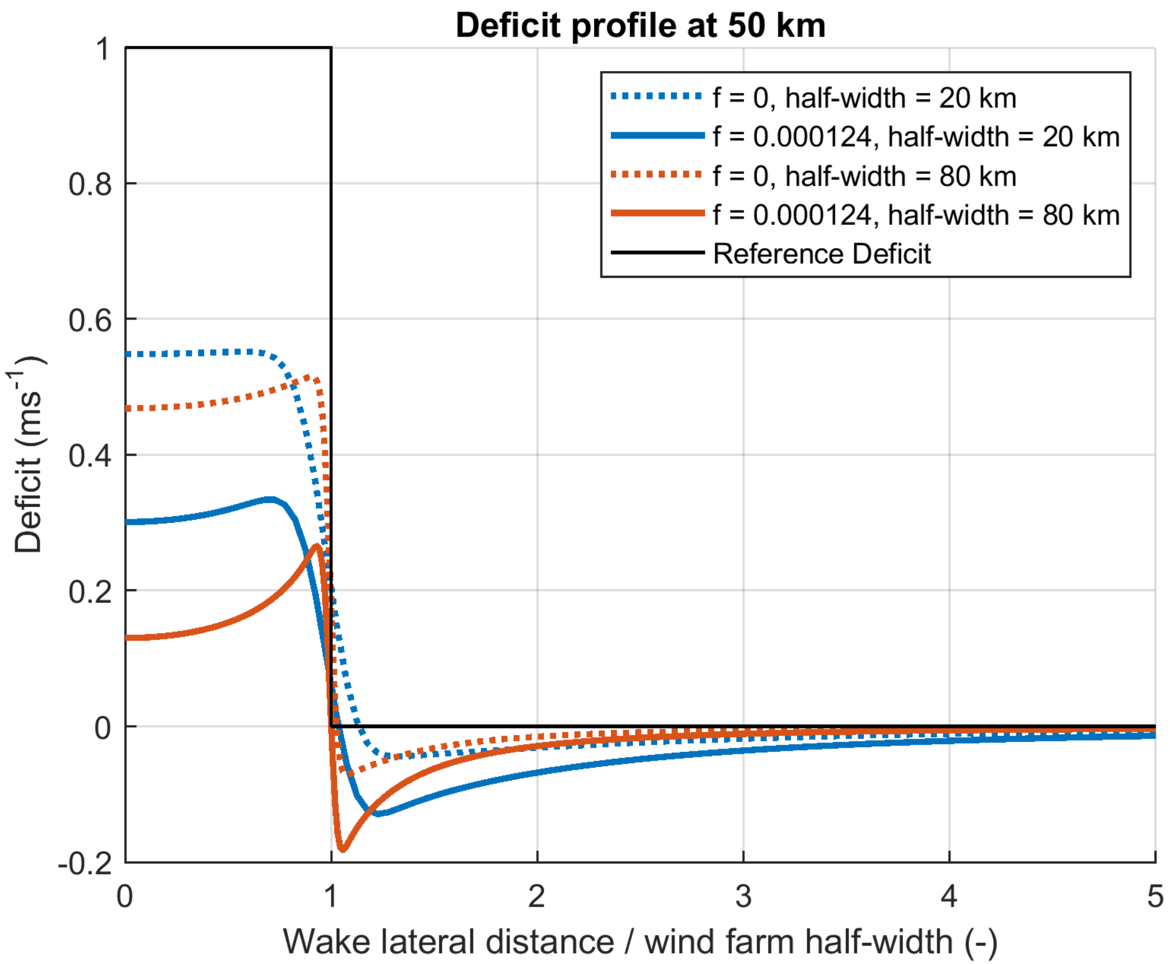
Red curve
Reference wake

[with Coriolis, f=0.00014 s^{-1}]



Special case of weak stability, weak mixing, wide farm, slow ambient wind, high latitude

Wake Deficit Profile 50 km downwind



Realistic Case:

$g' = 0$

$N = 0.01\ s^{-1}$

$C = 0.00005\ s^{-1}$

$f = 0.000124\ s^{-1}$

wind speed = 7 ms^{-1}

Coriolis has strong effect on inner wake recovery.
Stronger effect for wider wind farm.

Conclusions

- Many interacting physical processes
- Solutions found using Fourier Transforms
  - Fast flow field computations
  - Analytical solutions can be found in some cases

- Coriolis will contribute to Wake Recovery when:
  - Weak tropospheric stability (to reduce the effect of geostrophic balance)
  - Wide wind farm (to reduce the effect of geostrophic balance)
  - Weak wind (to give Coriolis more time to act)
  - Weak vertical mixing (to give Coriolis more time to act)
  - High latitude (stronger Coriolis force)

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