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# Long Range Wake Calculations Sensitive To Surface Layer Stability

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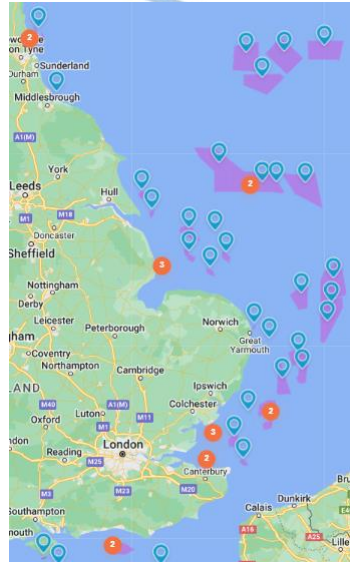
- 
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## The Challenge

- ▶ Current offshore wind farm map for UK →
- ▶ Not shown:
  - ▶ Other Countries
  - ▶ New Wind Farms
- ▶ Very crowded and will get more so!
- ▶ We need models to accurately predict yields, and to understand that accuracy
  - ▶ Familiar wind farm wake models are not applicable for long range wakes
- ▶ (The industry would prefer an ensemble of rapid models)



The Crown Estate Asset Map  
<https://www.thecrownestate.co.uk/en-gb/what-we-do/asset-map/>

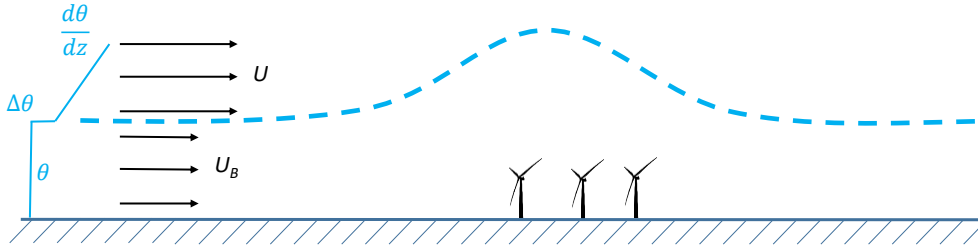
## Motivation and Approach

- ▶ At long range, we know that wakes merge
  - ▶ Individual wake resolution less important
- ▶ Long been postulated in Top-Down models that
  - ▶ For very long length scales, energy exchange with the atmosphere is what matters
  - ▶ This is sensitive to surface layer stability
- ▶ R B Smith formulation
  - ▶ Is very fast
  - ▶ Provides a link between turbine level, farm level and atmospheric processes
  - ▶ Can be made sensitive to surface layer stability
- ▶ Other rapid models
  - ▶ Have made really significant progress in past two years
  - ▶ May have some limitations due to not directly modelling ABL height and stability

## RB Smith Formulation



- ▶ 'Slab' model of Atmospheric Boundary Layer
  - ▶ Depth-averaged quantities



- ▶ Analytical solution for horizontal pressure field set up by ABL displacement
- ▶ Coupled to conservation of mass and momentum within ABL
- ▶ Closed equation set
- ▶ Linearisation around undisturbed condition permits very fast FFT solution

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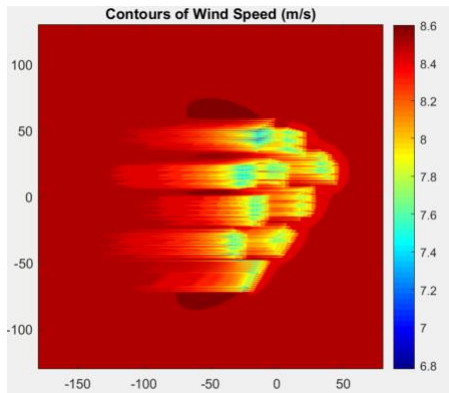
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## Add Horizontal Eddy Viscosity

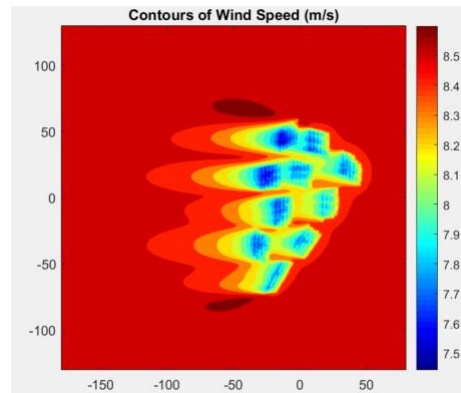


$$u_{B,0} \frac{\partial u'_B}{\partial x} + v_{B,0} \frac{\partial u'_B}{\partial y} = -\frac{1}{\rho_B} \frac{\partial p'_B}{\partial x} + \nu \frac{\partial^2 u'_B}{\partial y^2} + F_x - C_T u'_B - C_B u'_B$$

$$u_{B,0} \frac{\partial v'_B}{\partial x} + v_{B,0} \frac{\partial v'_B}{\partial y} = -\frac{1}{\rho_B} \frac{\partial p'_B}{\partial y} + \nu \frac{\partial^2 v'_B}{\partial x^2} + F_y - C_T v'_B - C_B v'_B$$



$\nu > 0$   
➔



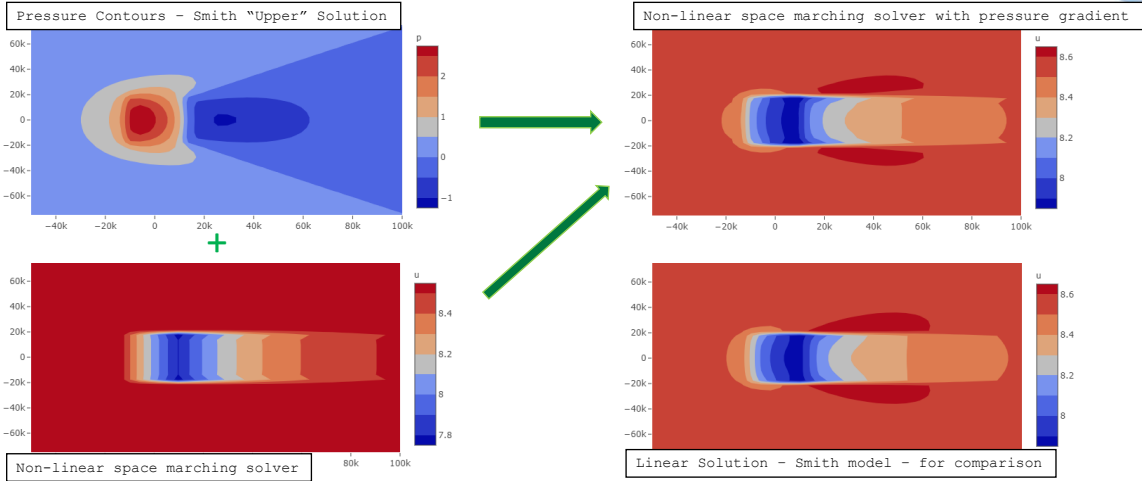
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### Space-Marching, Non-linear Solver



$$u_B \frac{\partial u_B}{\partial x} + v_B \frac{\partial u_B}{\partial y} = -\frac{1}{\rho_B} \frac{\partial p_B}{\partial x} + F_x + C_T(U - u_B) - C_B u_B$$



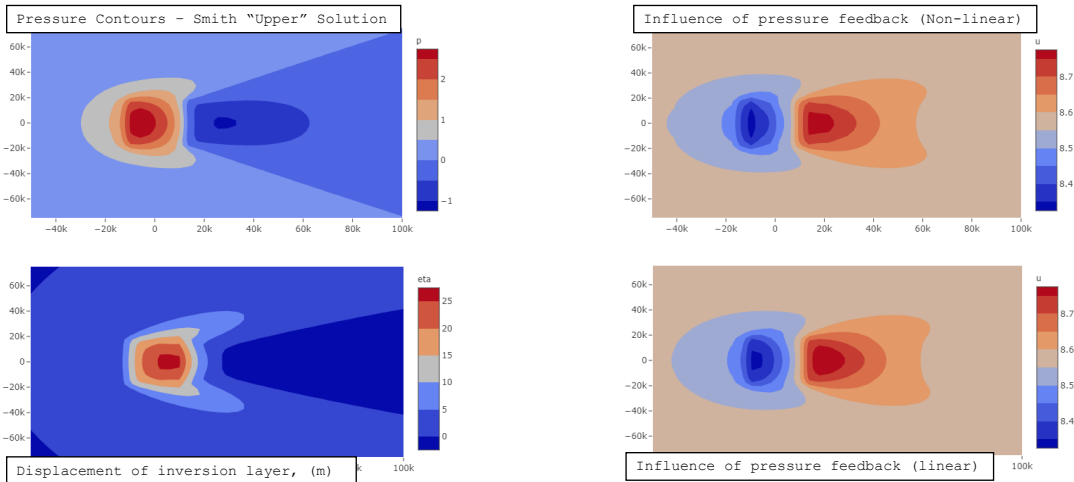
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### Side note : Influence of Pressure Feedback On Wind Speed



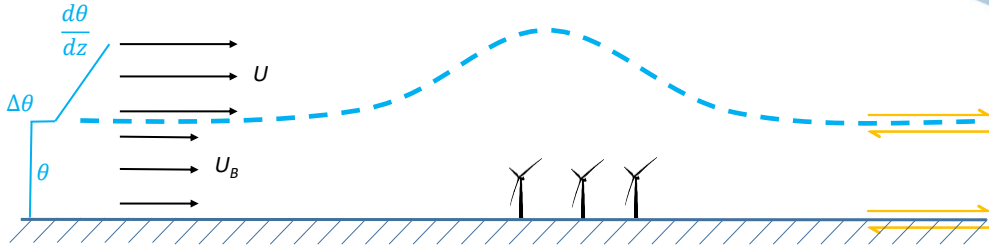
$$u_B \frac{\partial u_B}{\partial x} + v_B \frac{\partial u_B}{\partial y} = -\frac{1}{\rho_B} \frac{\partial p_B}{\partial x} + \overset{0}{F_x} + C_T(U - u_B) - C_B u_B$$



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### Rayleigh Friction Coefficient



$$u_B \frac{\partial u_B}{\partial x} + v_B \frac{\partial u_B}{\partial y} = -\frac{1}{\rho_B} \frac{\partial p_B}{\partial x} + \nu \frac{\partial^2 u_B}{\partial y^2} + F_x + C_T(U - u_B) - C_B u_B$$

$$u_{B,0} \frac{\partial u'_B}{\partial x} + v_{B,0} \frac{\partial u'_B}{\partial y} = -\frac{1}{\rho_B} \frac{\partial p'_B}{\partial x} + \nu \frac{\partial^2 u'_B}{\partial y^2} + F_x - C_T u'_B - C_B u'_B$$

$C_T = C_T \text{ (MOL)?}$   
 $C_B = C_B \text{ (MOL)?}$



### Rayleigh Friction Coefficient



$$C_B = 2 (u^*)^2 / (H u_B)$$

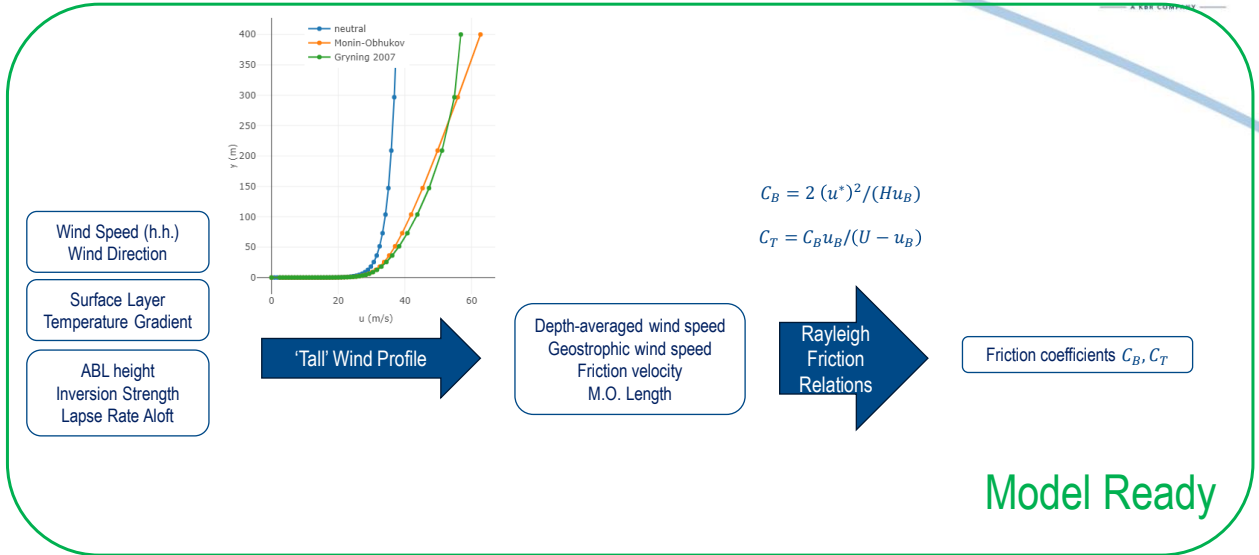
- ▶ RB Smith 2007
- ▶ Also, similar magnitudes by
  - ▶ Algebraic treatment of log law profile
  - ▶ Numerical treatment of 'tall' wind profile

$$C_T = C_B u_B / (U - u_B)$$

- ▶ RB Smith 2007 (shape function arguments)
- ▶ Also, same result from assuming top and bottom friction forces balance approximately



### Calculation Process



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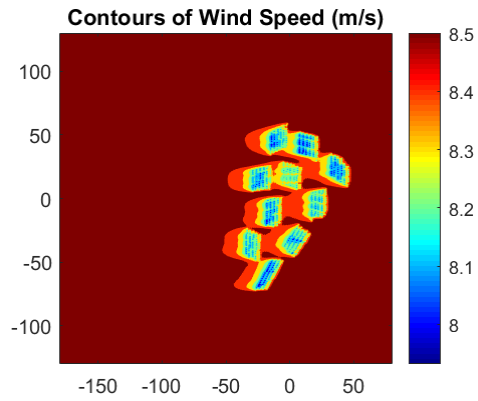
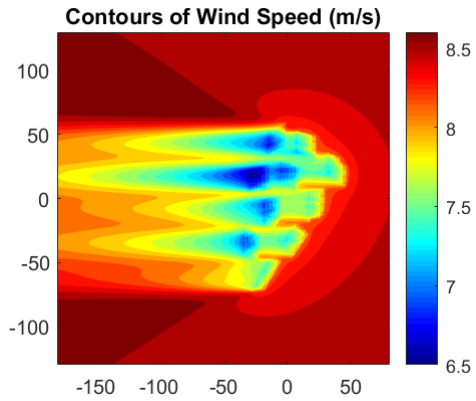
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### Example : Danish Energy Island Concept



STABLE: M.O.L. = + 50

UNSTABLE: M.O.L. = - 100



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## Validation

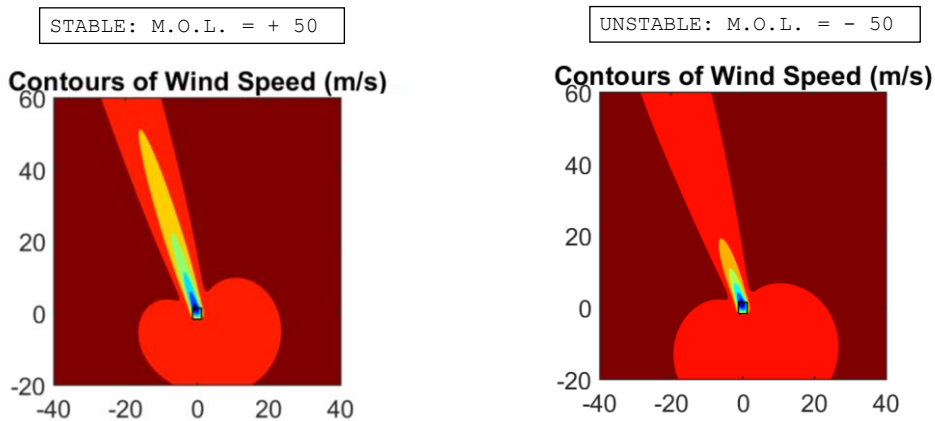


- ▶ Alpha Ventus wind farm wake length estimated via SAR
- ▶ Simultaneous measurement of wind speed and temperature gradient *in situ*
  - ▶ 'Impact of atmospheric stability on X-band and C-band synthetic aperture radar imagery of offshore windpark wakes', B. Djath, J. Schulz-Stellenfleth and B. Cañadillas. *J. Renewable Sustainable Energy* 10, 043301 (2018).
- ▶ Pioneering observational data set
- ▶ Rare opportunity for model validation

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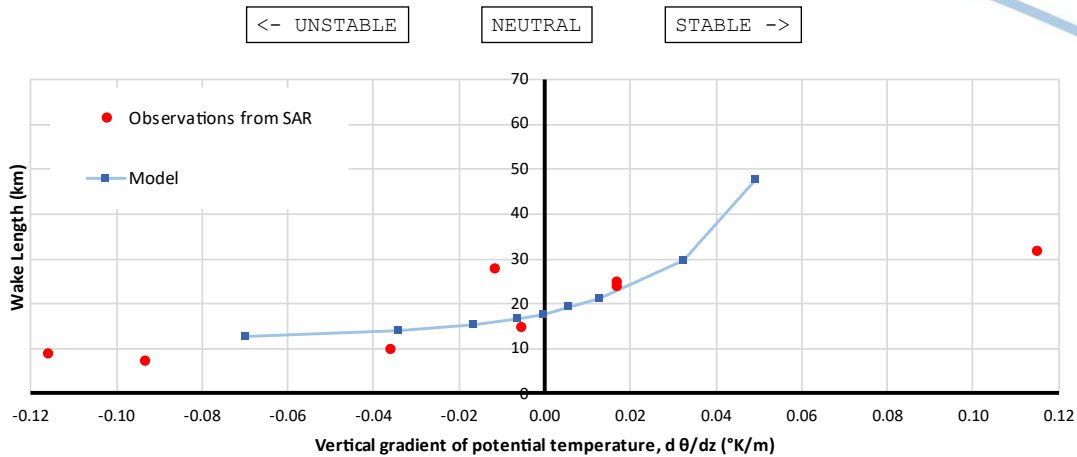
## Results



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## Results



Linear model results shown - Non-linear results very similar

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## Results & Findings

- ▶ R B Smith 'shallow layer' model applied to long range wakes problem
  - ▶ Rational and workable approach to surface layer sensitivity
  - ▶ Effective viscosity in horizontal plane added
  - ▶ Feasibility confirmed
  - ▶ Initial validation promising
  
- ▶ Strengths:
  - ▶ Directly represents stability on wake recovery
  - ▶ Very quick to run
  - ▶ Can use turbine thrust from standard models
  - ▶ Pressure field and non-linearity investigation
  
- ▶ Weakness:
  - ▶ Demands ABL/thermal profile data
  - ▶ Relies on 'Tall Wind Profile'
    - ▶ Hard to trust above 160m
  
- ▶ Thank you RAVE for pioneering measurement campaigns

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## References and Related Work



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3. "Analysis of Some Major Limitations of Analytical Top-Down Wind-Farm Models", Stefan Emeis, Boundary-Layer Meteorology, January 2022.
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7. 'On the extension of the wind profile over homogeneous terrain beyond the surface boundary layer', S-E Gryning, E Batchvarova, B Brümmer, H Jørgensen and S Larsen. Boundary-Layer Meteorology, Volume 124, March 2007.
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